HOF 01-079

本田財団レポートNo.79

「フラクタル，認識と印象の統合」

エール大学教授，IBM名誉フェロー　ブノワ・B・マンデルブロー

■Personal History

1924 Born in Warsaw, Poland.

1947 Ingénieur de I'Ecole Polytechnique, Paris, France.

1952 D. Sc in Mathematics, University of Paris, France.

1953～54 Member, Institute for Advanced Study, Princeton, N. J. USA.

1955～58 Junior Professor of Mathematics, Université de Genève. then Université de Lille and Ecole Polytechnique.

1958～93 Research Staff Member (IBM Fellow after 1974) IBM T. J. Watson Research Center, Yorktown Heights, N. Y. USA.

1987～ Abraham Robinson Professor of Mathematical Sciences, Yale University, New Haven Conn. USA.

● In addition, Dr. Mandelbrot has visited Harvard University repeatedly, as a Visiting Professor of Economics, later of Applied Physics and then of Mathematics, and has visited other institutions as well.

■Awards

1985 Barnard Medal for Meritorious Service to Science, "Magna est Veritas" : USA National Academy of Sciences and Columbia University.

1986 Franklin Medal for Signal and Eminent Service in Science: The Franklin Institute, Philadelphia Pa.

1988 Charles Proteus Steinmerz Medal: IEEE Chapter and General Electric Corporation, Schenectady N. Y.

1988 Senior Award (Humboldt Preis) : Alexander von Humboldt-Stiftung, Bonn, Germany.

1988 "Science for Art" Prize: Fondation Moët-Hennessy-Louis Vuitton, Paris.

1989 Harvey Prize for Science and Technology: Technion-Israel Institute of Technology, Haifa, Israel.

1993 Wolf Foundation Prize for Physics: Wolf Foundation of Israel to Promote Science and Art for the Benefit of Mankind.

● Excerpt from Wolf Prize citation "has changed our view of nature." He belongs to the U. S. National Academy of Sciences, to the American Academy of Arts and Sciences and the European Academy.

■Pubications

1982 The Fractal Geometry of Nature, W. H. Freeman & Co.

■略歴

1924 ポーランドのワルシャワに生まれる。

1947 フランス、エコール・ポリテクニックを卒業。

1952 フランス、パリ大学で学位(数学博士)を取得。

1953～54 アメリカ、プリンストン高等研究所研究員。

1955～58 ジュネーブ大学数学科助教授、リール大学及びエコール・ポリテクニック助教授を歴任。

1958～93 IBM・トーマス・J・ワトソン研究所研究員。(1974年よりIBMフェロー)

1987～ アメリカ、エール大学数学科教授。

● この他にハーバード大学で経済学、応用物理学及び数学の客員教授を務めたほか、その他の研究機関でも講師、客員教授を歴任した。

■主な受賞歴

1985 米国国立科学アカデミー及びコロンビア大学、バーナード・メダル。
「マグナ・エ・ヴェリタス」

1986 アメリカ、フランクリン研究所、フランクリン・メダル。

1988 アメリカ、IEEEチャプター・アンド・ジェネラル・エレクトリック社、チャールズ・プロテウス・シュタインメッツ・メダル。

1988 ドイツ、アレクサンダー・フォン・フンボルト財団、フンボルト賞。

1988 フランス、モエ・ヘネシー・ルイ・ヴィトン財団、「芸術科学」賞。

1989 イスラエル、テクニオン－イスラエル・先端技術研究所、ハービー科学技術賞。

1993 人類の利益のための科学及び芸術の促進を目指すイスラエル・ウルフ財団、ウルフ財団物理学賞。

● ウルフ財団物理学賞の表彰状には、「我々の自然に対する見解を変えた」との言葉が記されている。現在、米国国立科学アカデミー、米国科学芸術アカデミー、及びヨーロッパ・アカデミーに所属。

■代表的な著書

1982 The Fractal Geometry of Nature, W. H. Freeman & Co.
(日本語版：「フラクタル幾何学」広中平祐監訳　日経サイエンス社発行)

Fractals and The Unity of Knowing and Feeling

Lecture at the Conferring Ceremony on the 17th of November 1994, in Tokyo

Professor Benoit B. Mandelbrot

The Winner of the Honda Prize 1994
Abraham Robinson Professor of Mathematical Sciences, Yale University
IBM Fellow Emiritus, Thomas J. Watson Research Center

Your Excellencies, President of the Honda Foundation, Mrs. Honda, distinguished guests, the Honda Prize is an honor I shall treasure in a very special way, because it rewards a central aspect of my work. Indeed, I do not represent here one field and one country: I am a Frenchman born in Poland whose prize-winning work was done in the USA. And my only true intellectual home is the cross-disciplinary work that you have chosen for this award. Fractal geometry, which I originated, has affected several established fields, but I have never thought that it should become one itself. Thus, I spent all my life between well-organized entities, and it is legitimate to guess that your kind attention was drawn to the fact that my interests are tightly connected, and some concern ecology and others concern technology. Actually, my professional interests range even more widely, from art to mathematics, via a long roundabout route. (This leaves little room-and little need-for hobbies !)

Fractal geometry means many different things to different people. , But one of its meanings can be viewed as standing above the other: fractal geometry is a recent mathematical and graphic implementation of some very old and basic insights of our culture, and perhaps even of all cultures of mankind. Let us start with a problem that is implicitly set up in the Bible. In the King James Version, the first lines of Genesis inform us that:

"In the beginning God created the heaven and the earth.

And the earth was without form...

And God said, Let there be light: and there was light...

And God made the firmament... and it was so...

And God said, Let... the dry land appear: and it was so."

The existence of light eventually begat optics; the existence of a firmament begat astronomy; the existence of land begat geology, and other sciences arose in the same vein from many of the later lines of Genesis.

Yet, most aspects of the heaven and the earth were never made orderly. They remained without form: tohu va vohu in the original Hebrew.

The over-reaching goal of my scientific life can now be stated: I have spent it looking for elements of order in tohu va vohu. In due time, those elements became organized in a discipline I called fractal geometry.

From the sublime to the merely remarkable, take the painter Eugene Delacroix (1798-1853), and consider these words he wrote in the Revue Britannique in 1850.

"Swedenborg claims, in his theory of nature, ... that the lungs are composed of a number of little lungs, the liver of little livers, the spleen of little spleens, etc. Without being such a grand observer, I have noticed this truth for a long time. I have often said that the branches of a tree are themselves little trees; fragments of rocks are similar to masses of rocks, particles of earth to enormous piles of earth. I am persuaded that one would find a quantity of such analogies. A feather is composed of a million feathers."

Let us now continue with a few words by Edward Whymper (1840-1910), the first explorer to climb the Matterhorn and the author of the book Scrambles Amongst the Alps 1860-1869.

"It is worthy of remark that ... fragments of ... rock ... often present the characteristic forms of cliffs from which they have been broken ... Why should it not be so if the mountain's mass is more or less homogeneous? The same causes which produce the small forms fashion the large ones; the same influences are at work the same frost and rain give shape to the mass as well as to its parts."

These quotes by Delacroix and Whymper introduce a second basic theme, that of self-similarity.

Fractal geometry is a branch of learning, more precisely, of knowing and feeling, that I conceived and built around the above two intertwined threads of thought: disorder in nature and self-similarity.

In its fully formalized form, fractal geometry is an enterprise in mathematics whose primary purpose is to help physics, geophysics and other sciences.

But in the process of reaching this goal, fractal geometry has the very distinctive feature of putting enormous reliance upon the eye.

Human babies - contrary to kittens - are born with open eyes, but they must learn to see. Much of learning comes frome experience and requires no theory.

But in many cases, theory can be of great help, or at least of great influence, witness the following words by Paul Cézanne (1839-1906) in a letter to E. Bernard, April 15, 1904.

"Treat nature according to the cylinder, the sphere and the cone, with everything put in proper perspective, so that each side of an object or a plane is directed toward a central point"

It happens that I admire most paintings by Cézanne, but not the preceding statement. More specifically, I endorse unhesitatingly its general thrust, that one cannot see without a theory, hence different theories lead to different forms of art. But Cézanne's work has to me a disagreeable ring of old technology, My disagreement with his words was stated forcibly in the following lines, which open my book, The Fractal Geometry of Nature.

"Why is geometry often described as "cold" and "dry?" One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree.
Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."

"More generally, I claim that many patterns of Nature are so irregular and fragmented, that, compared with Euclid (an old English term I use to denote all of standard geometry) Nature exhibits not simply a higher degree but an altogether different level of complexity. The number of distinct scales of length of natural patterns is for all practical purposes infinite." "The existence of these patterns challenges us to study those forms that Euclid leaves aside as being "formless," to investigate the morphology of the "amorphous."....."

"Responding to this challenge, I conceived and developed a new geometry of nature and implemented its use in a number of diverse fields. It describes many of the irregular and fragmented patterns around us, and leads to fullfledged theories, by identifying a family of shapes I call fractals."

My own experience, confirmed by many stories that I heard, suggests that acquaintance with fractals makes humans see the world differently. This can happen at several different levels. When a recent physics award cited me for having "changed our view of nature," it referred to the view of nature as expressed in physicists' writings. But friends who are photographers tell me that fractals not only changed their view of nature, not in an allegorical sense, but in the most literal sense one could imagine.

As we shall see in a moment, fractal geometry also has, in addition to its realistic face, a face that is thoroughly non-representational. Fractals are a family of geometric shapes, and I happen to believe that, in order to understand geometric shapes, one must see them. It has very often been forgotten that geometry simply must have a visual component, and I believe that in many contexts this omission proved to be very harmful.

Fractal geometry is conveniently viewed as a language, and it has proven its value by its uses. Its uses in art and pure mathematics, being without "practical" application, can be said to be poetic. Its uses in various areas of the study of materials and of other areas of engineering are examples of practical prose. Its uses in physical theory, especially in conjunction with the basic eqations of mathematical physics, combine poetry and high prose.

Let me remind you of a marvellous text that Galileo Galilei wrote at the dawn of science, in his book, Il Saggiatore (1623) :

"Philosophy is written in this great book - I am speaking of the Universe - which is constantly offered for our contemplation, but which cannot be read until we have learned its language and have become familiar with the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles and other geometric forms, without which it is humanly impossible to understand a single word of it; without which one wanders in vain across a dark labyrinth."

We all know that mechanics and calculus, therefore all of quantitative science, were built on these characters, and we all know that these characters, belong to Euclidean geometry. In addition, we all agree with Galileo that this geometry is necessary to describe the world around us, beginning with the motion of planets and the fall of stones on Earth.

But is it sufficient? Figure 1 seems to represent a real mountain, but it is neither a photograph, nor a painting. It is a mathematical forgery, a computer forgery; it is completely based upon a mathematical formula from fractal geometry. The same is true of the forgery of a cloud shown in Figure 2.

Figure 1. A fractal landscape that never was (R. F. Voss).

Figure 2. A cloud formation that never was (S. Lovejoy & B. B. Madelbrot)

An amusing and important feature of Figures 1 and 2 is that both use new adaptations of formulae that had been known in pure math ematics. Thanks to fractal geometry, diverse mathematical objects, which used to be viewed as being so far from physics as to be "pathological", have turned out to be the proper tools for studying nature.

One of the successes of fractal modelling was unexpected and amusing. A fractal generator is used in *Star Trek Two, the Wrath of Khan.* The many people who saw this film witnessed a planet appear in the Genesis sequence, but few noticed without prodding that this planet is fractal. Prodded again, one sees peculiar characteristics (superhighways and square fields) that are due to a shortcut taken by Lucasfilm to make it possible to compute these fractals quickly enough. But we need not dwell on flaws. Far more interesting is the fact that the films that include fractals create a bridge between two activities that are not expected to ever meet, mathematics and physics on the one hand, and popular art on the other.

More generally, an aspect of fractals that I found very surprising at the beginning, and that continues to be a source of marvel, is that people respond to fractals in a deep, emotional fashion. They either like them or dislike them, and either emotion is completely at variance with the boredom that most people feel towards classical geometry.

Let me stop here to state that I will never say anything bad about Euclid's geometry. I love it and it has been an important part of my life as a child and as a student; in fact the main reason why I survived academically despite chaotic schooling was that I could always use geometric intuition to cover my lack of skill as a manipulator of formulae. Fractal shapes are exactly as geometric as those of Euclid, yet they evoke emotions which geometry is not expected or supposed to evoke.

Let us now move from the geometry of the world around us to the proper geometry of deterministic chaos: it happens to be the same as the proper geometry of mountains and clouds. The fact that we need only new geometry is really quite marvellous, because several might have been needed, in addition to that of Euclid. But it is not so. Fractal geometry plays both roles. Not only is it the proper language to describe the shape of mountains and of clouds, but it is also the proper language for all the geometric aspects of chaos.

To give an example, Figure 3 is an enormously magnified fragment from the set to which my name has been attached. Here, a

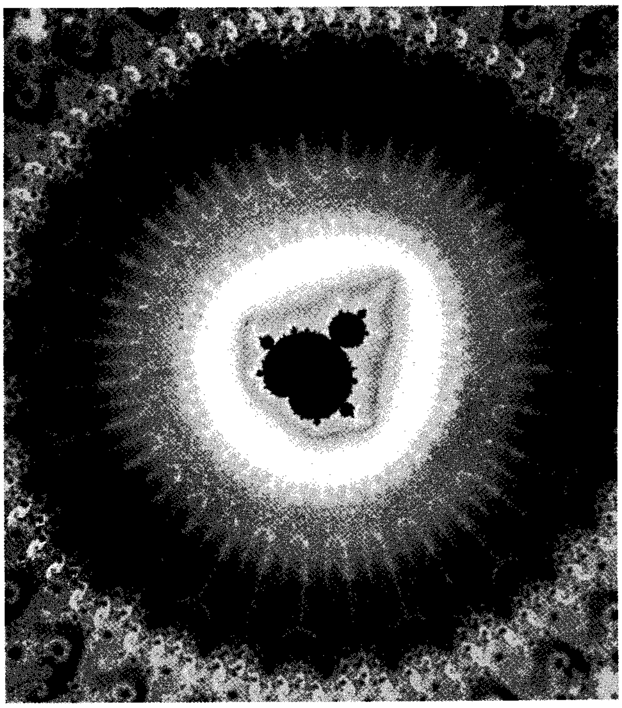

Figure 3. A very small fragment of the Mandelbrot Set (R. F. Voss).

fragment of the Mandelbrot Set has been magnified in a ratio equal to Avogadro's number, which is 24 decimal digits long. Why choose this particular number? Because it's a nice, very large number, and a huge magnification provided a good opportunity for testing the quadruple precision arithmetic on the I. B. M. computers that was being introduced some few years ago. (They passed the test. It's very amusing to be able to justify plan fun and pure science on the basis of down-to-earth considerations.) If the whole Mandelbrot Set had been drawn on the same scale, the end of it would be somewhere near the star Sirius.

The shape of the black "bug" near the centre is very nearly the same as that of the white centre of Figure 12, which shows the shape of the whole Mandelbrot Set. Finding nearly identical bugs all over the Set is a token of geometric orderliness. On the other hand, the surrounding pattern depends very much upon the point on which the zoom has focused; its variability is a token of variety, and even chaos.

The shape shown in Figure 4 is a variant of the Mandelbrot Set that corresponds to a slightly different formula. This shape is reproduced here simply to comment on a torally amazing and extraordinarily satisfying aspect of fractal geometry. Fractals are perceived by many people as being beautiful. But these shapes were initially developed for the purpose of science, for the purpose of understanding how the world is put together both staically (in terms of mountains) and dynamically (in terms of chaos, strange attractors, etc.). In other words, the shapes shown in Figures 1 to 4 were not intended to be beautiful. This being beautiful unavoidably raises many questions. The most important question is simply why? The fact must tell us something about our system of visual perception.

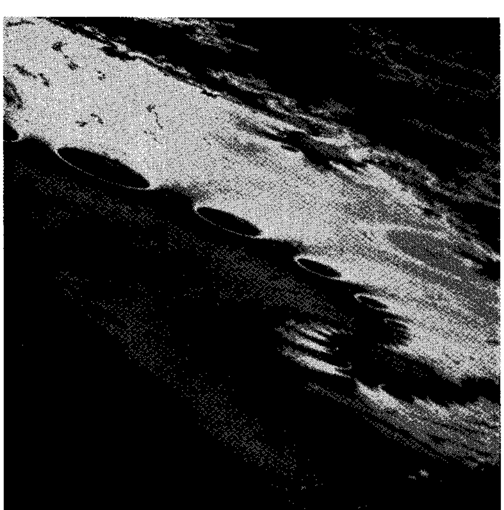

Figure 4. A small fragment of a modified Mandelbrot Set (B. B. Madelbrot)

I wanted to start with Figures 1 to 4 because their structure is so rich... but I went overboard. Their structure is in fact so rich that these figures cannot be used to explain the main feature of all fractals. The underlying basic principle shows far more clearly on

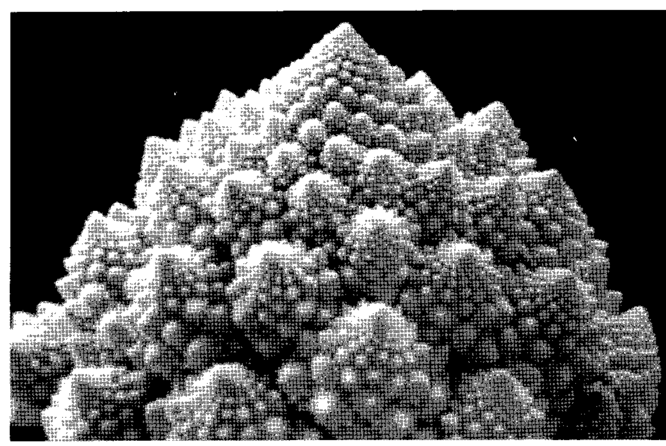

Figure 5. Cauliflower Romanesco(R. Ishikawa).

Figure 5, which -for a change- reproduces a real photograph of a real object. You may recognize a variety of cauliflower called Romanesco. Each bud looks absolutely like the whole head, each bud subdivides into smaller buds, and so on. I am told that the same structure repeats over the five levels of separation you can do by hand and see by the naked eye, and then over many more levels you can see only with a magnifying glass or microscope.

Until recently, scientists did not pay much attention to this "hierarchical" property. Their first reaction to this kind of botanical shape was not to focus on buds within buds, but on the spirals formed by the buds. This interest led to extensive knowledge about the relation between the golden mean (and the Fibonacci series), and the way plants spiral.

But the hierarchical structure of buds is more important for us here, because it embodies the essential idea of fractal.

Before we continue and tackle what a fractal is, let us ponder what a fractal is not. Take a geometric shape and examine it in increasing detail. That is, take smaller and smaller portions and enlarge each to some prescribed overall size. If our shape belongs to standard geometry, it is well known that the enlargements become increasingly smooth. In sharp contrast, the shapes I have been showing fail to be locally linear. In fact, they deserve being called "geometrically chaotic," unless proven otherwise. In an altogether different neighborhood of the great City of Science, a kind of geometric chaos became known during the half century from 1875 to 1925. Mathematicians who were attempting to flee from concern with nature became aware of the fact that a geometric shape's roughness need not vanish as the examination becomes more searching. It is conceivable that its roughness should either remain constant, or vary endlessly, up and down. The hold of standard geometry was so powerful, however, that the resulting shapes were not recognized as models of nature. Quite the contrary, they were labelled "monstrous" and "pathological." After discovering these sets, mathematics proceeded to increasingly greater generality.

Science must constanly navigate between two dangers: lack and excess of generality. Between the two extremes, it must always find the proper level that is necessary to do things right. Between the extremes of the excessive geometric order of Euclid, and of the true geometric chaos of the most general mathematics, can there be a middle ground of "organized" or "orderly" geometric chaos? To provide such a middle ground is the ambition of fractal geometry.

The reason why fractals are far more special than the most general shapes of mathematics is that they are characterized by transformations called "symmetries", which are invariances under dilations and/or contractions. Broadly speaking, mathematical and natural fractals are shapes whose roughness and fragmentation neither tend to vanish, nor fluctuate up and down, but remain essentially unchanged as one zooms in continually and examination is refind. Hence, the structure of every piece holds the key to the whole structure.

The preceding statement is made precise and illustrated by Figure 6, which represents a shape that is enormously simpler than Figures 1 to 5. As a joke, I called it the Sierpinski gasket, and the joke has stuck.

The four small diagrams show the point of departure of the construction, then its

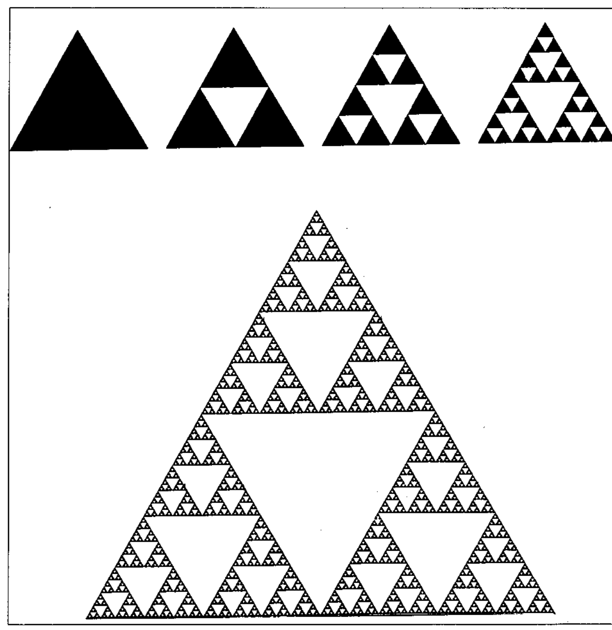

Figure 6. The Sierpinski gasket. Early and late stages of construction.

first three stages, while the large diagram shows an advanced stage. The basic step of the construction is to divide a given (black) triangle into four sub-triangles, and then erase (whiten) the middle fourth. This step is first performed with a wholly black filled-in triangle of side 1, then with three black triangles of side 1/2. This process continues, following a pattern called recursive deletion, which is very widely used to construct fractals.

By examining the large advanced stage picture, it is obvious that each of the three reduced gaskets is simply superposed on one third of the overall shape. For this reason, the fractal gasket is said to have property of exact or linear self-similarity.

I used to think that the word "self-similarity" was used for the first time in a paper of mine in 1964. But, it has since come to my attention that the philosopher Emerson (1803-1882) used it once. Now, why was this the old sense of the word. The two are very intimately linked.

(a)

(b)

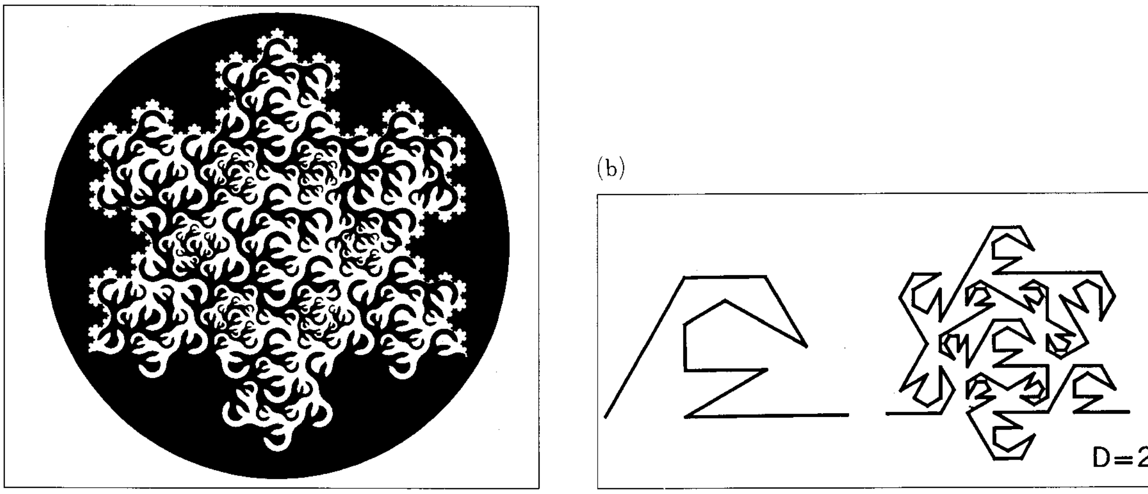

Figure 7. Mandelbrot's Peano curve. (B. B. Madelbrot). It is shown because it is attractive, but it is not referred to in this introduction.

word not used, although the idea itself is perfectly obvious and very old ? The reason is that finding that a shape is being self-similar had no importance until my work. For example, Sierpinski had investigated "his" shape for a long forgotten purpose for which the only virtue of self-similarity was that it resulted in a shape requiring few lines to describe.

Why did self-similarity become important ? Because Figures 1 to 5 are self-similar, not-to be sure-in an exact, but a statistical meaning of the word.

One reason why fractal geometry has developed so widely, and I spent so much time in efforts to build it as a discipline, resides in empirical discoveries (each established by a separate investigation) that the relief of the earth is self-similar, and that the same is true if many other shapes around us.

The Sierpinski gasket, and other structures of the same ilk, are important because you must begin the study of fractal geometry with them, but keep in mind that the real fun begins beyond them.

The fun begins after one has added an element of unpredictability, which may be due to either randomness (as in Figures 1, 2 and 5), or to non-linearity (as in Figures 3 and 4). Non-linearity is the key word of the new meaning of chaos, namely of deterministic chaos, and randomness is the key to chaos in

Figure 8 combines a sequence of completely artificial random landscapes. Each part of this picture consists of enlarging a small black rectangle in the preceding picture, and in filling in additional detail. This procedure is called recursive addition. Each step followed by "zooming-in" yields a landscape that is of course different from the preceding landscape. It is more detailed, yet at the same time is qualitatively the same. The successive enlargements might have been different parts of the same coastline examined on the same scale, but in fact they are neighborhoods of one single point examined very different scales. Clearly, these successive enlargements of a coastline completely fail to become locally smooth !

At this point, let me recall a story about the great difficulties the ancient Greeks used to experience in defining "bigness" in the context of geography.

Much evidence suggested Sardinia was less big than Sicily, but ancient sailors claimed that Sardinia was the bigger of the two: it took longer to circumnavigate, because its coastline was longer. But let us examine Figure 9, and ponder the notion of the length of a coastline ? When the ship used to circumnavigate is large, the captain will report a rather small length. A much smaller ship would come closer to the shore, and navigate along a longer curve. A man

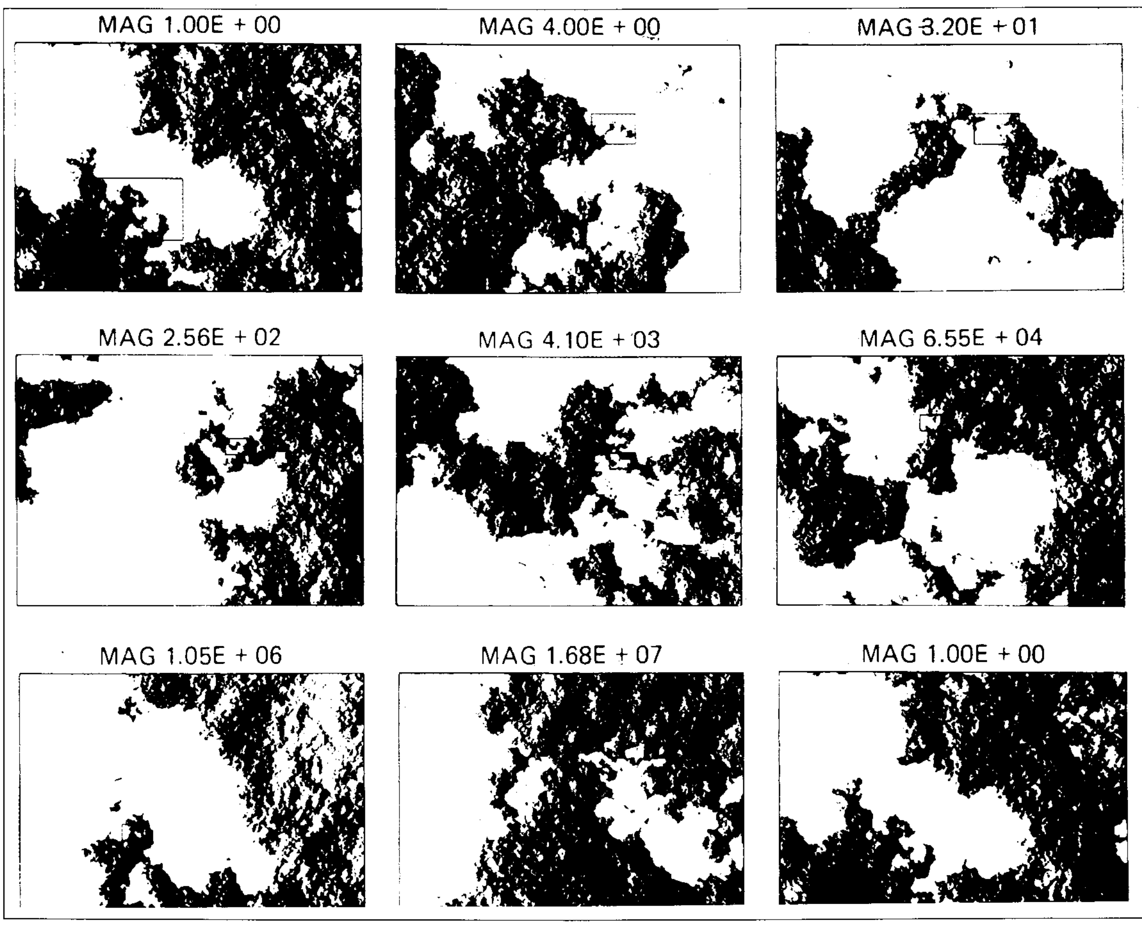

Figure 8. Zoom onto a fractal landscape that never was (R. F. Voss).

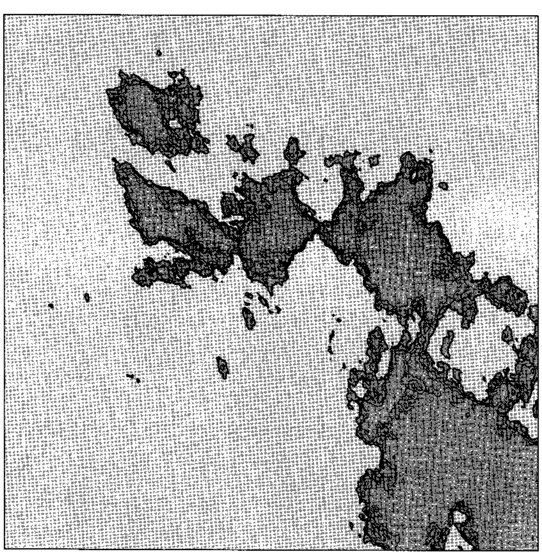

Figure 9. A fractal coastline that never was (B. B. Madelbrot)

walking along the coastline will measure an even longer length. So what about the "real length of the coast of Sardinia?" The question seems both elementary and silly, but it turns out to have an unexpected answer. The answer is, "there ia no single answer; it all depends." The length of a coastline depends on whether you circumnavigate in a large or a small ship, or walk along it, or use dividers of some other instrument to measure the coastline on a map.

The preceding example makes us appreciate the extraordinary power of the mental structure that schools impose by teaching Euclid. Many people who thought they had never understood geomerty learned enough to expect every curve to have a length. For the curves in which I am interested, this turns out to have been the wrong thing to remember from school, because the theoretical length is infinite, and the practical length depends on the method of measurement. Its increase is faster where the coastline is rough, making it necessary to study the notion of roughness.

The task of measuring roughness objectively has turned out to be extraordinarily difficult. People whose work demands it, like metallurgists, ask their friends in statistics for a number one could measure and call roughness. But perform the following experiment. Take different samples of steel which the Bureau of Standards guarantees to be pieces of one block of metal, as homogenous as man can make it. If you take several pieces and you break them all and measure the roughness of the fractures according to the books on statistics, you will get values that are in complete in disagreement

On the other hand, I shall argue that roughness happens to be measured consistently by a quantity called fractal dimension, which happens in general to be a fraction, and which one can measure very accurately. Studying many samples from the same block of metal, we found the same dimension for every sample.

The reason for the term "dimension" is that the same approach can also be applied to points, intervals, full squares and full cubes, and in those cases yields the familiar values 0, 1, 2 and 3. Applied to fractals however, measurements usually yield values that are not integers.

Fractal geometry has proved an increasingly valuable tool in the discovery and study of new aspects of nature. Diffusion Limited Aggregates, DLA, are a form of random growth. A DLA cluster lurks in the centre of Figure 10. It is a tree-like shape of baffling complexity one can use to model how ash forms, how water seeps through rock, how cracks spread in a solid and how lightning discharges.

To see how the growth proceeds, take a very large chess board and put a queen, which is not allowed to move, in the central square. Pawns, which are allowed to move in either of the four directions on the board, are released from a random starting point at the edge of the board, and are instructed to perform a random walk, or drunkard's walk. The direction of each step is chosen from four equal probabilities. When a pawn reaches a square next to that of the original queen, it transforms itself into a new queen and cannot move any further. Eventually, one has a branched, rather spidery-looking collection of queens.

Quite unexpectedly, massive computer simulations have shown that DLA clusters are fractal. They are nearly self-similar, that is, small portions are very much like reduced versions of large portions. But clusters deviate from randomized linear self-similarity, something that will pose interesting challenges for the future.

One reason for the importance of DLA is that it concerns the interface between the smooth and the fractal. Moving away again from randomness to deterministic chaos, and

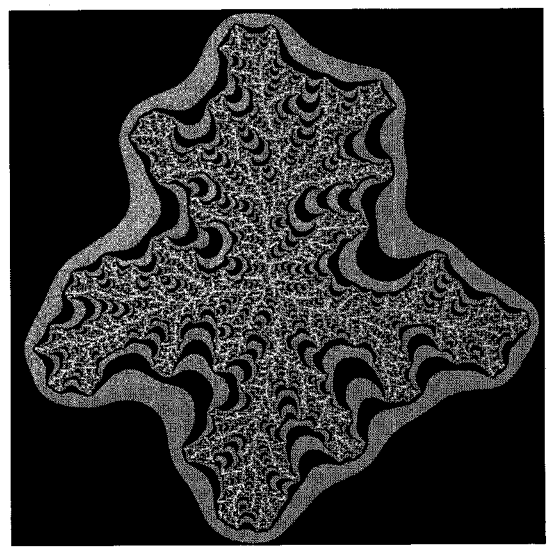

Figure 10. A cluster of diffusion limited aggregation, surrounded by its equipotential curves(C. J. G. Evertsz and B. B. Mandelbrot).

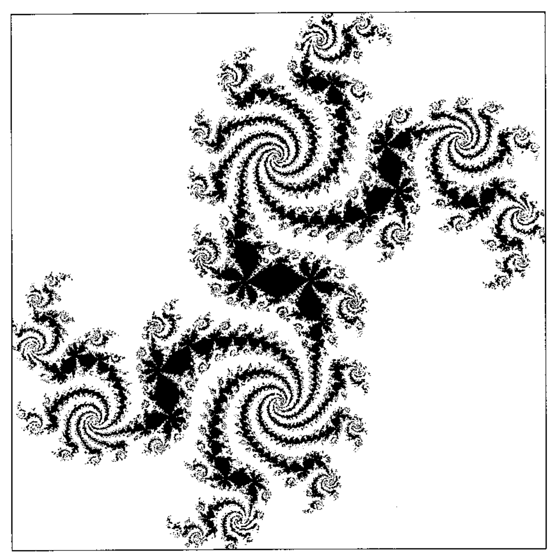

Figure 11. Quadratic Julia sets for the map Z. Z+C. Each boundary of zebra stripe corresponds to a different value of C (B. B. Mandelbrot).

from physical to imaginary objects, let us consider Julia sets. What will remain unchanged is that we shall deal with spiky sets surrounded by smooth lines.

An example of a "filled-in Julia set" is shown in Figure 11. This is generated by iteration of the simple function Z . Z+C. Iteration means that the result of each evaluation provides the starting point to the next evaluation; because Z and C may be complex numbers, negative values can occur. For starting points outside the black shape, iteration will go to infinity; if you start inside, you fail to iterate to infinity. The boundary between black and white is called the Julia curve. It is approximately self-similar. Each chunk is not quite identical to a bigger chunk, because of non-linear deformation. But it is astonishing that iteration should create any form of self-similarity, quite spontaneously.

As in the investigation of fractal mountains, the computer was essential to the study of iteration. The bulk of fractal geomerty is concerned with shapes of great apparent complication and by hand they could never be drawn. More precisely, this picture could have been computed by a hundred people working for years; but nobody would have started such an enormous calculation, without first feeling that it was worth performing.

Not only had I access to a computer in 1979, but I was familiar with its capabilities. Therefore, I felt these calculations were worth trying, even though I certainly did not know what was going to come out. A fishing expedition led to a primitive form of Figure 12. The Julia sets of the map Z. Z+C can take all kinds of shapes, and a small change in C can change the Julia set very greatly. I set out to classify all the possible shapes and came up with a new shape, that has come to be known as the Mandelbrot Set, M. Figure 3 shows a tiny portion of Figure 12.

As you zoom towards a portion of the boundary of M, part of that you see is simply a repetition of something you have already seen. This element of repetition is essential to beauty. But beauty also requires an element of change and that is also very

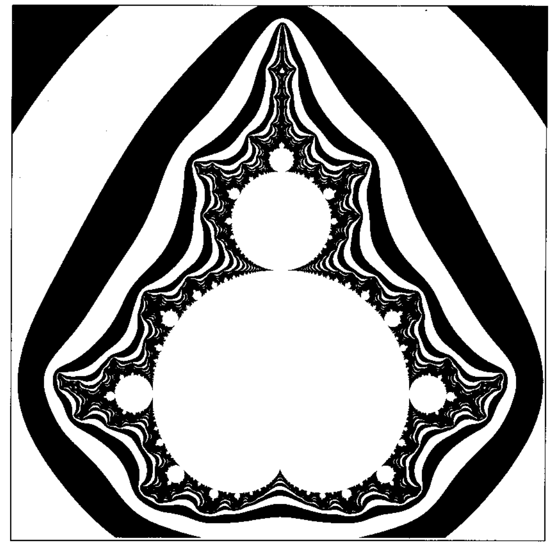

Figure 12. The Mandelbrot Set, surrounded by its equipotential curves.

clearly present. As you come closer and closer, what you see becomes more and more complicated. The overall shape is the same, but the hair structure becomes more and more intense. This feature is not something we put in on purpose. Insofar as mathematics is not invented but discovered. It is something that has been there forever and it shows that the mathematics of Z. Z+C is astonishingly complicated, by contrast with the simplicity of the formula. We find that the M set, when examined closer and closer and closer, exhibits the coexistence of the relentless repetition of the same theme combined with variety that boggles the imagination. I first saw the Mandelbrot Set on a black and white screen of very low graphic quality, and the picture looked dirty. But when we zoomed on what seemed like dirt we found instead an extraordinary little copy of the whole.

In Figure 12, the Mandelbrot Set is the white "bug" in the middle. It is very rough-edged, but is surrounded by a collection of zebra stripes whose edges become increasingly smooth as one goes away from M. These zebra stripe edges happen to be Laplacian equipotential curves - just like in Figure 10. But they are far easier to obtain.

Of course, the black and white figures in this introduction are far from the beautiful color ones which everyone must have seen. The structure itself is independent of the color rendering. However, different renderings emphasize very different structural aspects. This use of color is similar to that employed in relief maps, where altitude bands are signified by different colors. Perhaps surprisingly, the black and white of Bill Hirst's photographs serves to clarify their structual content.

Let me now bring together the separate strings of this presentation. How did fractals come to play their role of "extracting order of chaos ?" The key resides in the following very surprising discovery I made thanks to computer graphics.

The algorithms that generate fractals are typically so extraordinarily short, as to look positively dumb. This means they must be called "simple." Their fractal outputs, to the contrary, often appear to involve structures of great richness. A priori, one would have expected that the construction of complex shapes would necessitate complex rules.

What is the special feature that makes fractal geomerty perform in such an unusual manner ? The answer is very simple. The algorithms are recursive, and the computer code written to implement them involves "loops." That is, the basic instructions are simple, and their effects can be followed easily.

But let these simple instructions be performed repeatedly and - unless one deals with the simple old fractals, such as the Sierpinski gasket the process of iteration effectively builds up an increasingly complicated transform, whose effects the mind can follow less and less easily. Eventually, one reaches something that is "qualitatively" different from the original building block. One can say that the situation is a fulfilment of what in general is nothing but a dream: the hope of describing and explaining "chaotic" nature as the cumulation of many simple steps.

Having surveyed some features of fractal geometry, I shall conclude by bringing back some theme of my introduction. Several parts of my work have been rewarded in the past, either singly or in various combinations, but I feel that this Honda Prize is directed very specifically at a broad cross-disciplinary theme that underlies all that I have done in my professional life. Once again, I shall treasure it in a very special way, and wish to express my deep gratitude to the persons and the institution that have brought it about.

Thank you very much.

The Fractal Geometry of Nature by B.B. Mandelbrot, W.H. Freeman, 1982 was the first comprehensive book on the subject, and remains a basic reference book.